Optimizing Nonlinearities in Neural Coding

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Every we know about the world around us is represented in the nervous system by sequences of discrete electrical pulses termed action potentials or "spikes". One attractive theoretical idea, going back to the 1950s, is that these representations constructed by the brain are efficient in the sense of information theory. These ideas have been formalized to predict the spatial and temporal filtering properties of neurons, as well as the shapes of nonlinear input/output relations, showing how these measured behaviors of cells can be understood as optimally matched to the statistical properties of natural sensory inputs. Despite this progress, relatively little attention has been given to the problem of optimal coding in the presence of the strong, threshold–like nonlinearities associated with the generation of spikes.

Sensory inputs to the brain are intrinsically high dimensional objects. For example, visual neurons encode various patterns of light intensities that, upon moderate discretization, become vectors in 10^2-10^3 dimensional space. We can think of the "decision" to generate an action potential as drawing boundaries in these high dimensional spaces, so that a theory of optimal coding for spiking neurons is really a theory for the shape of these boundaries. One might think that the simplest and most stable way to encode stimuli is to use planar decision boundaries (Figure 1). Indeed, no transmission is perfect. With most errors occurring near the decision boundary, planar boundaries would have the smallest length, and therefore could potentially be more reliable. However this argument ignores that different stimuli occur with different frequencies (gray color scale in Figure 1). This is especially pronounced for signals typical of the natural sensory environment.

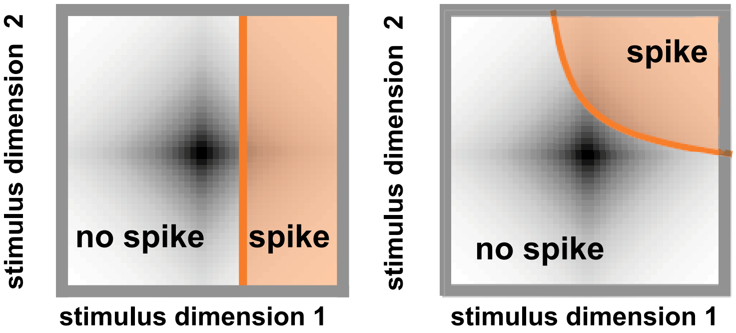

Figure 1. Various threshold-like strategies that neurons could use to encode multi-dimensional stimuli. Color shows how often stimuli are distributed, with darker colors for more frequent stimuli. Each neuron can separate stimuli into two categories, responding with a "spike" in the voltage trace across the neuron's membrane to some stimuli and not responding to others. Even when the average spike probability is pre-determined, there are still multiple ways how a given neuron can assign its "spike" to stimuli. However, no transmission is perfect, and errors are bound to occur. Most of the uncertainty would come from near the decision boundary. Careful placement of this decision boundary relatively to most frequent stimuli could minimize errors and maximize information transmission. In some cases, including those that approximate statistics of signals in the natural sensory environment, optimal decision contours can be curved (see Figure 2), even though one might think that the simplest solution is to use the planar boundaries as in the left panel in this figure.

Last year Sharpee and Bialek [1] have shown that while Gaussian inputs, a favorite theoretical approximation, are optimally separated by planar boundaries, this is not the case in general. In particular, neural encoding of exponentially distributed inputs in two dimensions is most reliable when the decision contours are curved and could either be closed or extended. Such inputs capture the statistical character of large-amplitude deviations from the mean value common in natural visual or auditory stimuli. The ubiquity of non–Gaussian signals in nature, particularly of the exponential distributions considered here, suggests that these results might be relevant for neurons across different sensory modalities.

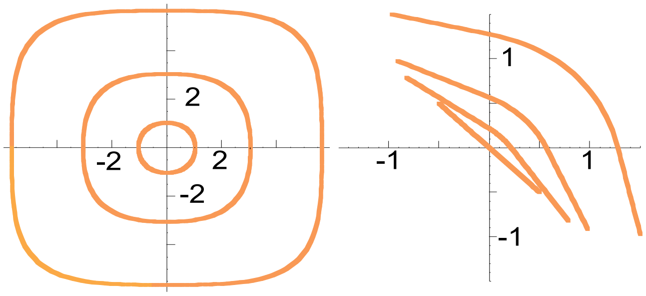

Figure 2. Shapes of optimal neural response regions for encoding 2D exponential inputs. These inputs capture the frequency of large-amplitude deviations from the mean typical of natural visual or auditory signals. Closed contours (left) are optimal at extreme probabilities, while extended ones (right) are optimal for spike probabilities near 1/2.

1. Sharpee, TO and W Bialek. Neural Decision Boundaries for Maximal Information Transmisssion. PloS One, 2007.